AEDC-TR-81-12

Dahlem
Newman
FILE

Comments on the Use of Measurements of the Effective Moment Parameter $\left(C_{\overline{m}_\alpha}\right)$

C. J. Welsh
Calspan Field Services, Inc.

August 1981

Final Report for Period October 1978 – February 1981

ARNOLD ENGINEERING DEVELOPMENT CENTER
ARNOLD AIR FORCE STATION, TENNESSEE
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

APPROVAL STATEMENT

This report has been reviewed and approved.

ALVIN R. OBAL, Captain, CF
Project Manager
Directorate of Technology

Approved for publication:

FOR THE COMMANDER

Marion L. Laster

MARION L. LASTER
Director of Technology
Deputy for Operations

REPORT DOCUMENTATION PAGE — READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AEDC-TR-81-12	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): COMMENTS ON THE USE OF MEASUREMENTS OF THE EFFECTIVE MOMENT PARAMETER $\left(C_{\overline{m}_\alpha}\right)$	5. TYPE OF REPORT & PERIOD COVERED: Final Report - October 1978 - February 1981
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): C. J. Welsh, Calspan Field Services, Inc.	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Arnold Engineering Development Center/DOT Air Force Systems Command Arnold Air Force Station, TN 37389	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Program Element 65807F
11. CONTROLLING OFFICE NAME AND ADDRESS: Arnold Engineering Development Center/DOS Air Force Systems Command Arnold Air Force Station, TN 37389	12. REPORT DATE: August 1981
	13. NUMBER OF PAGES: 19
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18 SUPPLEMENTARY NOTES

Available in Defense Technical Information Center (DTIC).

19 KEY WORDS (Continue on reverse side if necessary and identify by block number)

angle of attack
nonlinear analysis
wind tunnel tests
curve fitting
computer programs

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An examination was made of the utilization of the effective moment parameter, $C_{\overline{m}_\alpha}$, that has been used for many years, in relation to a recently devised procedure that involves the concept of an effective angle of attack for the nonlinear motion history being analyzed. This procedure provides an approximate direct relationship between a $C_{\overline{m}_\alpha}$ measurement and its parent nonlinear moment curve, for a large family of nonlinearities. Both the

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

20. ABSTRACT (Continued)

additional use of $C_{\overline{m}_\alpha}$ values and the restraints in their utilization are discussed in relation to small amplitude, large amplitude, and out-of-plane modes of testing.

AFSC
Arnold AFS Tenn

UNCLASSIFIED

PREFACE

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by Calspan Field Services, Inc., operating contractor for the Aerospace Flight Dynamics testing effort at the AEDC, AFSC, Arnold Air Force Station, Tennessee. The Air Force project manager was Mr. Marshall Kingery. This work was done under AEDC project numbers V32S-P4 and V-45G. The manuscript was submitted for publication on June 10, 1981.

CONTENTS

ILLUSTRATIONS

1.0 INTRODUCTION

An appreciable capability exists in the use of curve-fitting computer programs for obtaining static stability constants from motion histories of bodies having forces and moments that are nonlinear with angle of attack. Such motion histories can be obtained in either aeroballistic ranges or wind tunnels. However, as demonstrated in Ref. 1, there are situations for which it is advantageous to obtain fits of "nonlinear motion histories"* by using curve-fitting programs in which only linear terms of the motion equations are used, thereby providing linear fits of the motion histories. When only linear terms are used in fitting nonlinear motion histories, mean static moment and force parameters are obtained. These are referred to as "effective" stability derivatives which are designated $C_{\overline{m}_\alpha}$ and $C_{\overline{N}_\alpha}$, respectively, in this report. For simplicity, only the effective moment derivative will be discussed here, but comments made will also apply to the corresponding effective force derivative.

Because the effective moment derivative, $C_{\overline{m}_\alpha}$, is measured over an interval of a motion history and hence corresponds to a band of motion amplitude, defining the relationship of $C_{\overline{m}_\alpha}$ with the corresponding nonlinear moment coefficient, C_{m_t}, has been a problem area in the past. C_{m_t} is the desired parameter from experimental tests of nonlinear configurations, and measurements of C_{m_t} are usable when they are defined either as a function of the angle of attack or tabulated versus corresponding angle-of-attack values.

An approximate procedure for relating $C_{\overline{m}_\alpha}$ to the corresponding nonlinear moment coefficient, using an effective angle-of-attack concept, was presented recently in Ref. 1. This procedure was shown to be very useful in the analysis of the nonlinear experimental data of Ref. 1. The purpose of this report is to discuss, with the use of the effective angle-of-attack concept of Ref. 1, the use of effective moment derivative measurements obtained by various investigators at different test conditions, (e.g. Refs. 2 to 8). Other approximate techniques such as those described in Refs. 9 and 10 have been published that concern relating a measured effective moment derivative to its corresponding nonlinear moment coefficient. Those techniques involved more general approaches to the problem and do not permit an assessment of the restrictions related to the direct utilization of experimental effective derivatives.

---

*Motion histories corresponding to bodies having nonlinear force and moment characteristics.

2.0 DISCUSSION

2.1 EFFECTIVE ANGLE-OF-ATTACK CONCEPT

The procedure presented in Ref. 1 for approximating the relationship of an effective moment derivative, $C_{\overline{m}_\alpha}$, to its corresponding nonlinear moment coefficient is briefly described in the following paragraphs.

For an ideal linear configuration, the moment coefficient can be written

$$C_{m_t} = C_{m_\alpha} \cdot \alpha \tag{1}$$

where C_{m_α} is a constant for different values of α. If the oscillatory motion histories of such a linear configuration are fitted with a computer program (using only the linear terms of the program), C_{m_α} of Eq. (1) is one of the parameters defined. The amplitude of the motion interval fitted is of no concern because C_{m_α} is a constant and C_{m_t} can be defined for the α values of interest.

For a configuration having a nonlinear moment that can be described with a cubic nonlinearity (a cubic nonlinearity is normally assumed in the fitting technique), then

$$C_{m_t} = C_{m_{1\alpha}} \cdot \alpha + C_{m_{3\alpha}} \cdot \alpha^3 \tag{2}$$

Here, α is used as the angle of attack (small angles) or sine of the angle of attack. If a motion history of such a configuration is fitted as above, using only the linear terms, then the motion-fitting program will define a quasi-linear moment parameter, $C_{\overline{m}_\alpha}$. The usefulness of $C_{\overline{m}_\alpha}$ values, as noted in Section 1.0, has been limited in the past by a lack of techniques for utilization of corresponding amplitude information available from the motion fit to deduce nonlinear moment characteristics. For example, if $C_{\overline{m}_\alpha}$ measurements were obtained from motion fits of a given configuration at different amplitude levels and those measurements were approximately constant, then such measurements would have confirmed that the moment variation for the configuration was approximately linear. However, if different values were obtained for $C_{\overline{m}_\alpha}$, then one would know only that the configuration had a nonlinear moment and that C_{m_t}, which is of primary concern in a nonlinear system, could not be defined directly.

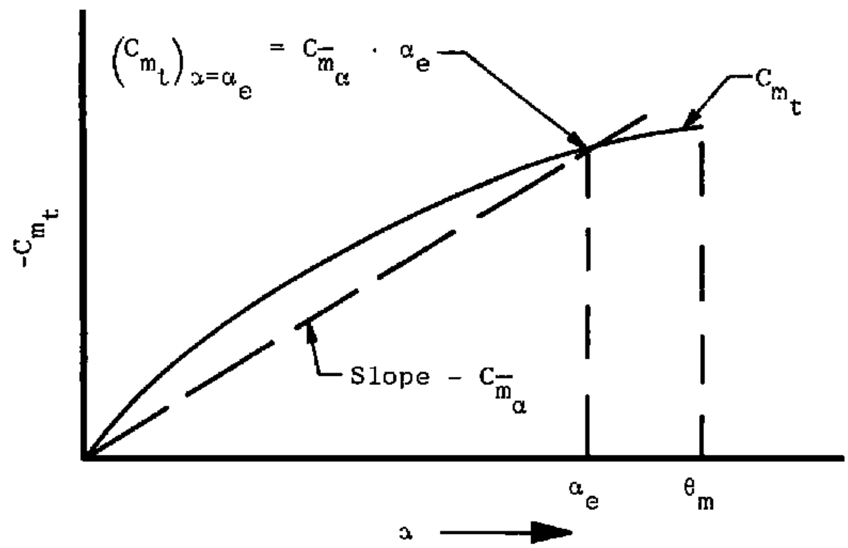
If a $C_{\overline{m}_\alpha}$ value has been determined from a fit of an interval of motion history of a vehicle that, in fact, has a nonlinear moment coefficient defined by Eq. (2), then it follows

that there is some effective angle of attack, α_e, of the motion interval fitted that corresponds to the $C_{\overline{m}_\alpha}$ such that C_{m_t} of Eq. (2) can be written

$$\left(C_{m_t}\right)_{\alpha = \alpha_e} = C_{\overline{m}_\alpha} \cdot \alpha_e = C_{m_{1\alpha}} \cdot \alpha_e - C_{m_{3\alpha}} \cdot \alpha_e^3 \tag{3}$$

The obvious problem here is in relating α_e to the amplitude of the motion interval fitted. To accomplish this, the ratio (α_e/θ_m) is used. Here, θ_m is the average of (a) the midrange amplitude of the fitted interval of motion, and (b) the mean of the initial and final motion amplitudes of the fitted interval. Further, θ_m corresponds to the major axis in the case of an elliptic angle-of-attack motion pattern. If α_e can be defined adequately, then one point on the nonlinear C_{m_t} curve can be defined for each fitted $C_{\overline{m}_\alpha}$ value without the problem of evaluating both $C_{m_{1\alpha}}$ and $C_{m_{3\alpha}}$ of Eq. (2).

The results of a study of the α_e concept presented in Ref. 1 for realistic free-flight conditions indicate that the (α_e/θ_m) ratio tends to be insensitive to the magnitude of the nonlinearity, to the type of motion pattern, and to the amplitude level involved for a fairly wide range of test conditions. The study utilized a series of computer-generated nonlinear motion histories for given $C_{m_{1\alpha}}$ and $C_{m_{3\alpha}}$ values which were fitted (linear fits utilizing the correct roll rates) to provide corresponding $C_{\overline{m}_\alpha}$ values. The α_e/θ_m ratio was shown to be nearly constant with a mean value of 0.87, and the corresponding C_{m_t} values can be obtained quite well using the α_e concept from nonlinear free-flight motion histories, given the following restrictions that (1) the test configuration is statically stable at zero angle of attack, (2) the magnitude of the nonlinear C_{m_t} curve corresponding to the maximum amplitude of the motion interval being fitted is not less than about 0.9 of the C_{m_t} peak value of the interval, (3) the ellipticity* of the motion pattern is less than about 0.5, and (4) the model roll rate is small. The relationship between $C_{\overline{m}_\alpha}$, C_{m_t}, α_e, and θ_m is shown below.



*Ratio of the lengths of the minor axis to the major axis of the elliptic motion pattern experienced by a body.

2.2 LARGE AMPLITUDE MODE OF TESTING

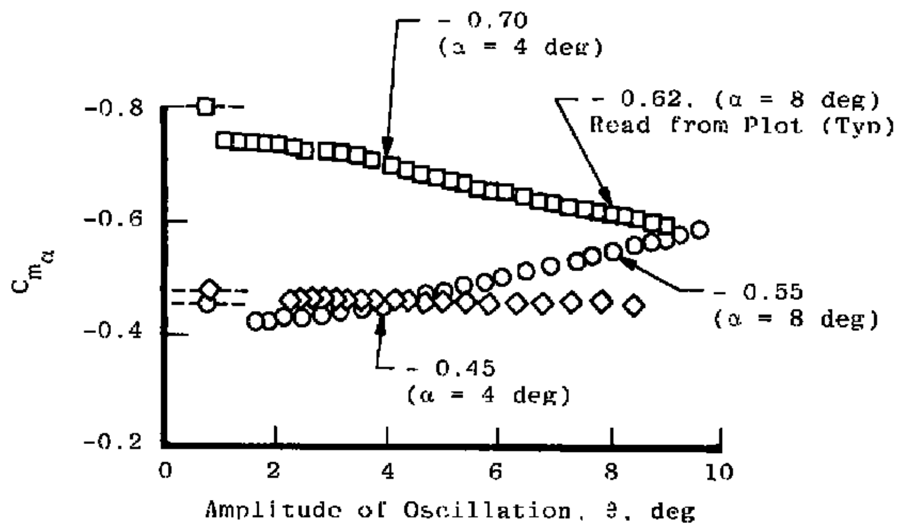
To demonstrate the limited use that has been made of large amplitude moment data, the measurements of $C_{\overline{m}_\alpha}$ of Fig. 2 of Ref. 2 have been replotted in Fig. 1a. These are measurements for cones with different nose bluntness ratios, as obtained with a large amplitude gas bearing pivot system (planar motion); they are representative of measurements of a series of large amplitude tests that have been conducted. Corresponding predicted moment coefficient curves (Ref. 11) for the two blunt cones are shown in Fig. 1b. These curves have appreciable nonlinearities.

Sym	r_n/r_b	x_{cg}/ℓ	X_{cg}/L	$\omega d/2V_\infty$	$Re_\ell \times 10^{-6}$	
◇	0.001	0.600	0.600	0.0009	2.42	In Ref. 2
□	0.151	0.542	0.600	0.0015	3.19	L - Virtual Cone Length
○	0.300	0.465	0.600	0.0012	2.72	ℓ - Actual Cone Length

$M_\infty = 10$

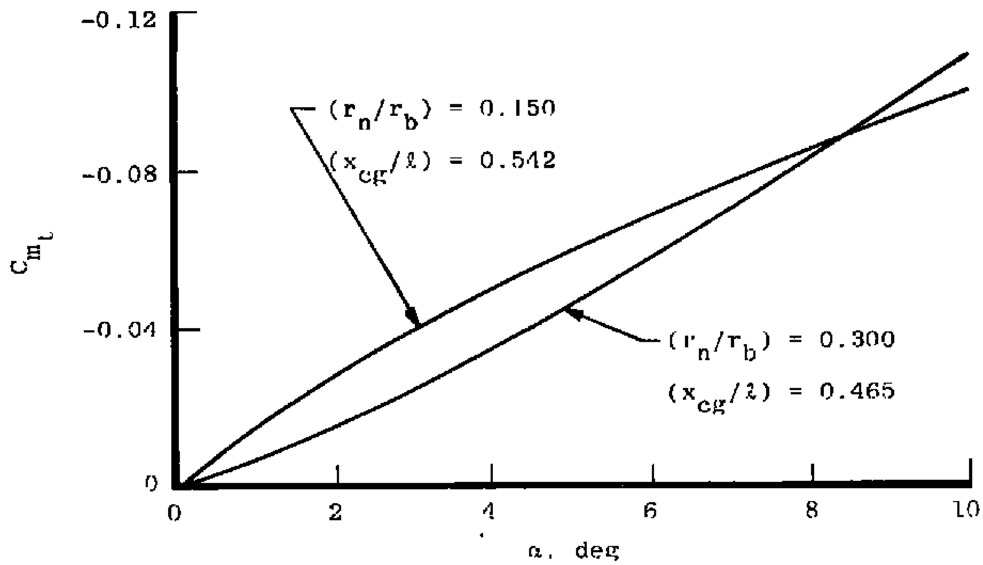
——— Unsteady Flow Field Theory

Note: C_{m_α} of Ref. 2 is same as $C_{\overline{m}_\alpha}$ of present report.



a. Large amplitude measurements (Fig. 2, Ref. 2)
Figure 1. Moment data for 10-deg semiangle cones.

As previously discussed, in testing nonlinear configurations it is the C_{m_t} values that are of primary concern. Reference 2 notes that the $C_{\overline{m}_\alpha}$ measurements were not constant with increasing amplitude for the two blunt cones, indicating that the corresponding moment coefficient curves would not be linear. This was an appropriate comment because a direct relationship between a $C_{\overline{m}_\alpha}$ measurement and its corresponding C_{m_t} value was not available at that time.



b. Predicted curves, Ref. 11 (M = 10)
Figure 1. Concluded.

In Fig. 2 the predicted moment curves of Fig. 1b are shown again, along with experimental C_{m_t} values computed using $C_{\overline{m}_\alpha}$ measurements of Fig. 1a at amplitudes of 4 and 8 deg and utilizing the effective angle-of-attack concept [with $(\alpha_e/\theta_m) = 0.87$]. The agreement of the predicted curves with the experimental C_{m_t} values in Fig. 2 provides additional confidence in the use of the effective angle-of-attack concept for obtaining nonlinear moment coefficients from $C_{\overline{m}_\alpha}$ measurements. A well-defined experimental C_{m_t} curve can be obtained by using additional $C_{\overline{m}_\alpha}$ measurements from Fig. 1a. The $C_{\overline{m}_\alpha}$ measurements of Ref. 2 would be expected to be of high quality because they were obtained with an air bearing pivot system which would have had a trivially small tare stiffness parameter. Normally, the measuring error in $C_{\overline{m}_\alpha}$ measurements in carefully designed tests can be kept to within one percent.

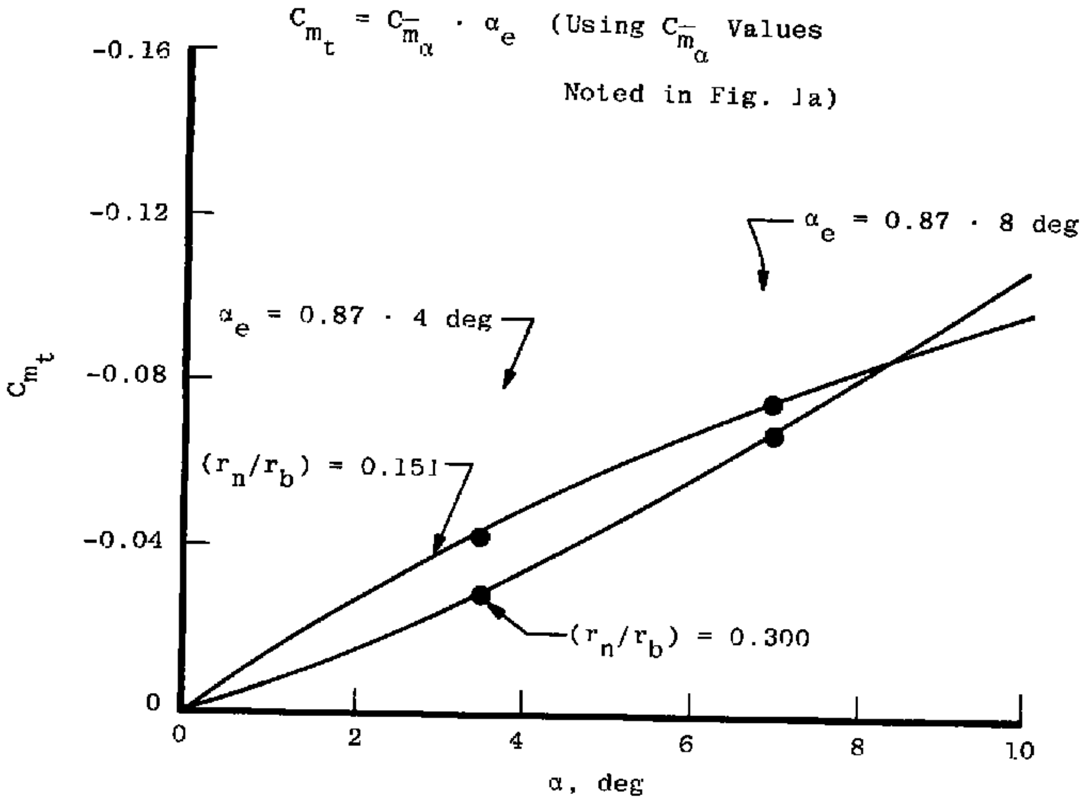
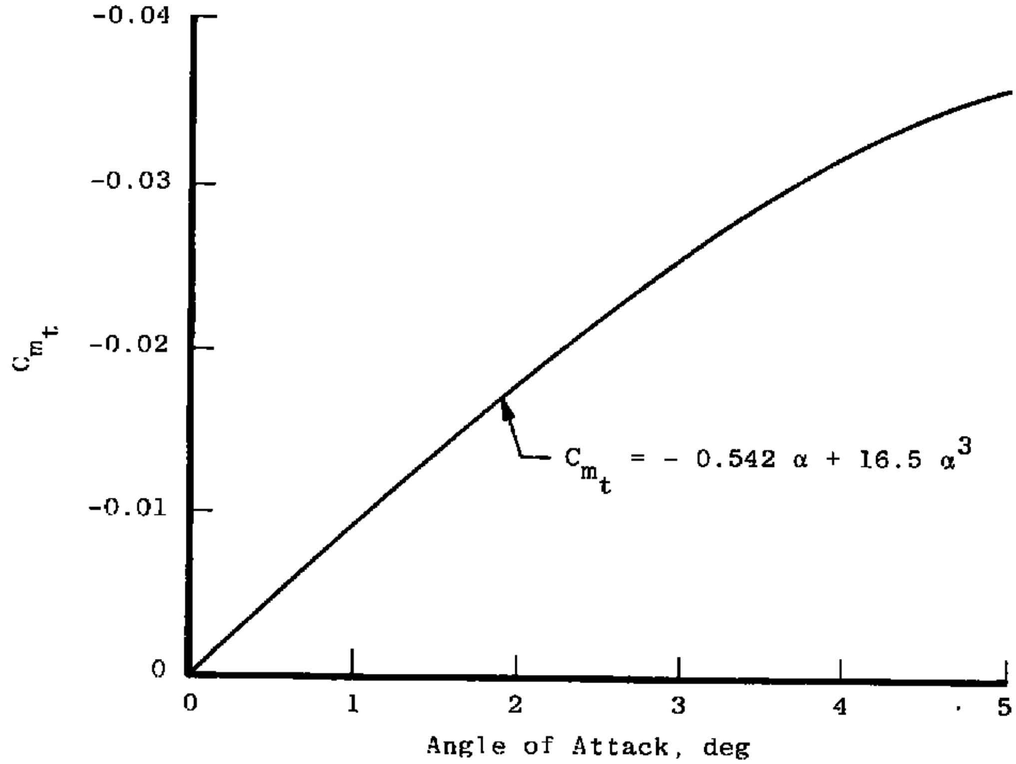


Figure 2. Comparison of C_{m_t} values computed from $C_{\bar{m}_\alpha}$ data of Fig. 1a with predicted curves of Fig. 1b.

2.3 SMALL AMPLITUDE MODE OF TESTING

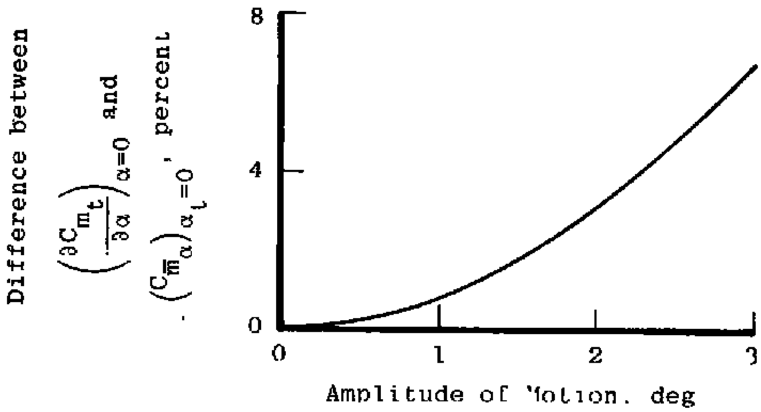
2.3.1 For $\alpha_t = 0$

In this mode of testing, previous investigators (see, for example, Refs. 3 and 4) have made $C_{\bar{m}_\alpha}$ measurements at amplitude levels between ± 1 and ± 3 deg. Because of the lack of information in the use of $C_{\bar{m}_\alpha}$ measurements, it has usually been assumed that $(C_{\bar{m}_\alpha})_{\alpha_t = 0}$ is equal to $(\partial C_{m_t}/\partial \alpha)_{\alpha=0}$. The validity of such an assumption is questionable, as demonstrated by the differences between $(\partial C_{m_t}/\partial \alpha)_{\alpha=0}$ and $(C_{\bar{m}_\alpha})_{\alpha_t=0}$ values shown in Fig. 3a. The curve of Fig. 3a corresponds to a configuration having a representative nonlinear moment curve (cubic nonlinearity with $C_{m_{1\alpha}} = -0.542$ and $C_{m_{3\alpha}} = +16.5$, as shown in the sketch on p. 11). This same assumed moment curve is used in assessing measurements made under other test conditions, as discussed in later sections of this report.



The curve in Fig. 3a was obtained from a computer study in which linear fits of generated nonlinear motion histories were obtained. Figure 3a shows that the difference between $(C_{\overline{m}_\alpha})_{\alpha_1 = 0}\,(\partial C_{m_t}/\partial\alpha)_{\alpha = 0}$ increases with the amplitude level of the test and is about seven percent at an amplitude of ±3 deg. Hence, to maximize the usefulness of $C_{\overline{m}_\alpha}$ measurements, it is important to always minimize the amplitude level used in experimental tests involving nonlinear configuration. This comment follows in consideration that testing can involve configurations having nonlinearities appreciably greater than that of the demonstration C_{m_t} curve shown above. Any nontrivial measuring error associated with the particular test equipment used could be an additional factor in selecting the optimum minimum test amplitude.

Note: $(C_{\overline{m}_\alpha})_{\alpha_t=0}$ values obtained from linear fits of nonlinear motion histories -- cubic function with $C_{m_{1\alpha}} = -0.542$ and $C_{m_{3\alpha}} = +16.5$.



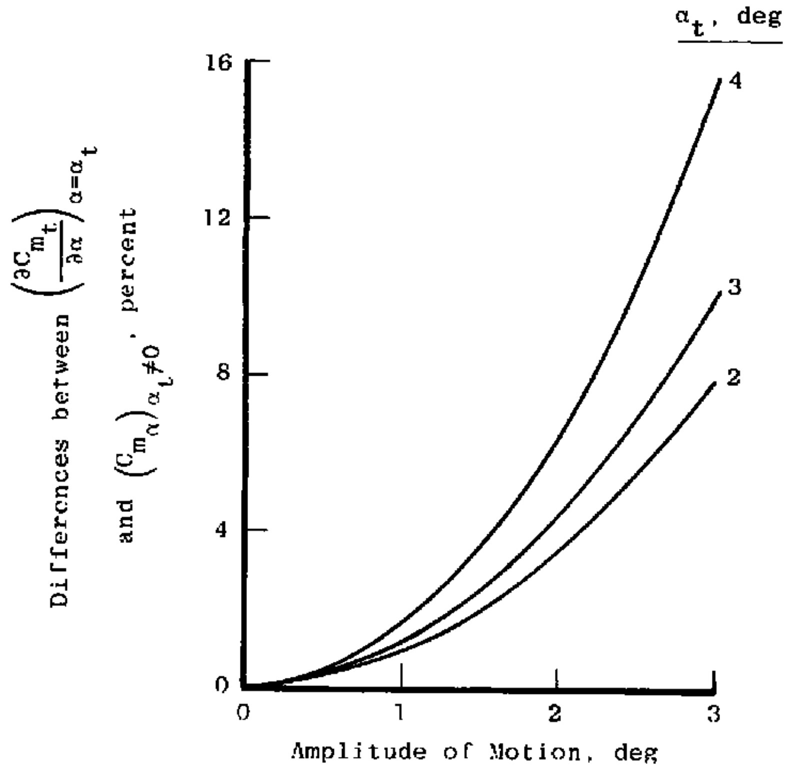
a. $\alpha_t = 0$

Figure 3. Comparisons of $C_{\overline{m}_\alpha}$ and $\partial C_{m_t}/\partial\alpha$ values.

2.3.2 For $\alpha_t \neq 0$

In this mode of testing, investigators in the past (see, for example, Refs. 4 through 8) have used a measured $(C_{\overline{m}_\alpha})_{\alpha_t \neq 0}$ as being equal to $(\partial C_{m_t}/\partial\alpha)_{\alpha=\alpha_t}$ again because of a lack of amplitude information for corresponding $C_{\overline{m}_\alpha}$ measurements. Differences between $(C_{\overline{m}_\alpha})_{\alpha_t \neq 0}$ and $(\partial C_{m_t}/\partial\alpha)_{\alpha=\alpha_t}$ that can be expected for a representative nonlinearity (same as used above) are shown in Fig. 3b. The curves in Fig. 3b were determined again in a computer study in which linear fits of generated nonlinear motion histories were obtained. In this case a pivot system with a tare restoring moment (having linear characteristics) was simulated using $(C_{\overline{m}_\alpha})_{pivot} = -3.0$; this permitted providing a trim moment to the model and, in turn, a trim angle when the support sting was set at a nonzero angle of attack. The curves of Fig. 3b indicate that the difference between $(C_{\overline{m}_\alpha})_{\alpha_t \neq 0}$ and $(\partial C_{m_t}/\partial\alpha)_{\alpha=\alpha_t}$ in small amplitude testing increases appreciably as the trim angle is increased. Again in such tests it is important to minimize the amplitude level used. It is significant to observe for a body having rotational symmetry that a nonlinear C_{m_t} curve corresponding to small amplitude motion about $\alpha_t \neq 0$ is asymmetric, and the use of $(\alpha_e/\theta_m) = 0.87$, derived for oscillatory motion about $\alpha_t = 0$, is not valid. Further, one can expect $(\alpha_e/\theta_m)_{\alpha_t \neq 0}$ to be an appreciably more sensitive parameter than $(\alpha_e/\theta_m)_{\alpha_t = 0}$. Because of the differences between $(C_{\overline{m}_\alpha})_{\alpha_t \neq 0}$ and

$(\partial C_{m_t}/\partial \alpha)_{\alpha=\alpha_t}$, which can be significant, one must exercise care in the use of a procedure utilized in the past in which a C_{m_t} curve is determined from an integration involving the corresponding $(C_{\overline{m}_\alpha})_{\alpha_t \neq 0}$ measurements.



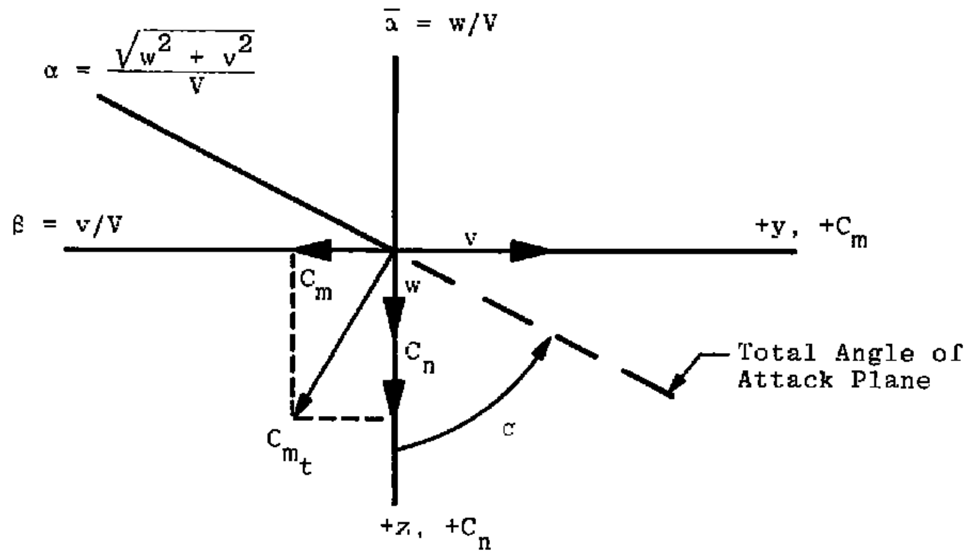
b. $\alpha_t \neq 0$
Figure 3. Concluded.

2.4 OUT-OF-PLANE MEASUREMENTS

In this mode of testing (see, for example, Refs. 6 through 8) the model has a trim angle of attack in one plane and small amplitude motion in the opposite plane. As an example, a typical set of test conditions could be $\alpha_t \neq 0$ and $\beta = \pm 2$ deg. For this mode of testing, it is useful to examine the corresponding moment equations involved for a body having rotational symmetry. For such a body the equation for the total moment coefficient, C_{m_t}, corresponding to a cubic nonlinearity was defined previously (Eq. 2):

$$C_{m_t} = C_{m_{1\alpha}} \cdot \alpha + C_{m_{3\alpha}} \cdot \alpha^3$$

Components of α are defined below.



Note: y and z are two axes of an orthogonal axis system that has its x-axis aligned with the longitudinal axis of the body.

Here the orientation of the total angle-of-attack plane relative to the orthogonal stability axes (y and z) is a function of the angle σ, which is defined by the velocity components v and w such that $\tan \sigma = v/w$. For stability, the total moment coefficient C_{m_t}, is oriented in the sketch consistent with the given direction of α. Note that in conventional motion equations, C_m and C_n are the desired components of C_{m_t} relative to the y and z axes and will have opposite signs. They can be written

$$C_n = C_{m_t} \cdot \sin \sigma \tag{4}$$

$$C_m = -C_{m_t} \cdot \cos \sigma \tag{5}$$

From symmetry considerations for a body with rotational symmetry where C_{m_t} is independent of σ and considering the sign difference between C_m and C_n noted above, the equation constants of Eq. (1) can be written in terms of the conventional stability coefficients as

$$C_{m_{1\alpha}} = C_{n_{1\beta}} = -C_{m_{1\bar{\alpha}}} \tag{6}$$

and

$$C_{m_{3\alpha}} = C_{n_{3\beta}} = -C_{m_{3\bar{\alpha}}} \tag{7}$$

Equations (4) and (5) expressed as functions of the velocity ratios can be written

$$C_n = \left[C_{n_{1\beta}} \frac{\sqrt{w^2 + v^2}}{V} + C_{n_{3\beta}} \left(\frac{\sqrt{w^2 + v^2}}{V} \right)^3 \right] \frac{v}{\sqrt{w^2 + v^2}} \tag{8}$$

$$= C_{n_{1\beta}} \left(\frac{v}{V}\right) + C_{n_{3\beta}} \left(\frac{v}{V}\right)^3 + C_{n_{3\beta}} \left(\frac{w}{V}\right)^2 \cdot \left(\frac{v}{V}\right)$$

$$= C_{n_{1\beta}} \beta + C_{n_{3\beta}} \beta^3 + C_{n_{3\beta}} \bar{\alpha}^2 \beta$$

and

$$C_m = \left[C_{m_{1\bar{\alpha}}} \frac{\sqrt{w^2 + v^2}}{V} + C_{m_{3\bar{\alpha}}} \left(\frac{\sqrt{w^2 + v^2}}{V} \right)^3 \right] \frac{w}{\sqrt{w^2 + v^2}} \tag{9}$$

$$= C_{m_{1\bar{\alpha}}} \left(\frac{w}{V}\right) + C_{m_{3\bar{\alpha}}} \left(\frac{w}{V}\right)^3 + C_{m_{3\bar{\alpha}}} \left(\frac{v}{V}\right)^2 \left(\frac{w}{V}\right)$$

$$= C_{m_{1\bar{\alpha}}} \cdot \bar{\alpha} + C_{m_{3\bar{\alpha}}} \bar{\alpha}^3 + C_{m_{3\bar{\alpha}}} \beta^2 \bar{\alpha}$$

The partial derivaties of C_n and C_m [Eqs. (8) and (9)] relative to β and α, respectively, are

$$\frac{\partial C_n}{\partial \beta} = C_{n_{1\beta}} + 3C_{n_{3\beta}} \beta^2 + C_{n_{3\beta}} \bar{\alpha}^2 \tag{10}$$

and

$$\frac{\partial C_m}{\partial \bar{\alpha}} = C_{m_{1\bar{\alpha}}} + 3C_{m_{3\bar{\alpha}}} \bar{\alpha}^2 + C_{m_{3\bar{\alpha}}} \beta^2 \tag{11}$$

These equations are instructive in that Eqs. (8) and (10) demonstrate that, even for a body with rotational symmetry, both the moment coefficient, C_n, and its corresponding partial

derivative, $(\partial C_n/\partial \beta)$, are dependent on the angular motion of the model in the opposite plane. A similar comment could be made regarding Eqs. (9) and (11).

It follows from Eqs. (10) and (11) that the in-plane partial derivative $(\partial C_m/\partial \bar{\alpha})_{(\beta=0,\bar{\alpha}=\bar{\alpha}_t)}$ can differ appreciably from the out-of-plane partial derivative $(\partial C_n/\partial \beta)_{(\beta=0,\bar{\alpha}=\bar{\alpha}_t)}$, and an example of such possible differences is shown in Fig. 4. The effective stiffness parameter, $(C_{\bar{n}_\beta})_{(\beta_t=0,\bar{\alpha}=\bar{\alpha}_t)}$, measured in out-of-plane tests can differ from $(\partial C_n/\partial \beta)_{(\beta=0,\bar{\alpha}=\bar{\alpha}_t)}$ and differences that can be expected for the same assumed representative nonlinearity, utilized previously, are shown in Fig. 5. The curves in Fig. 5 again demonstrate the importance of always minimizing the motion amplitude that is used in such tests.

From these comments concerning out-of-plane measurements, it follows that an investigator can have a dilemma in assessing the usefulness of $(C_{\bar{n}_\beta})_{(\beta_t=0,\bar{\alpha}=\bar{\alpha}_t)}$ type measurements in the general case of an asymmetric body.

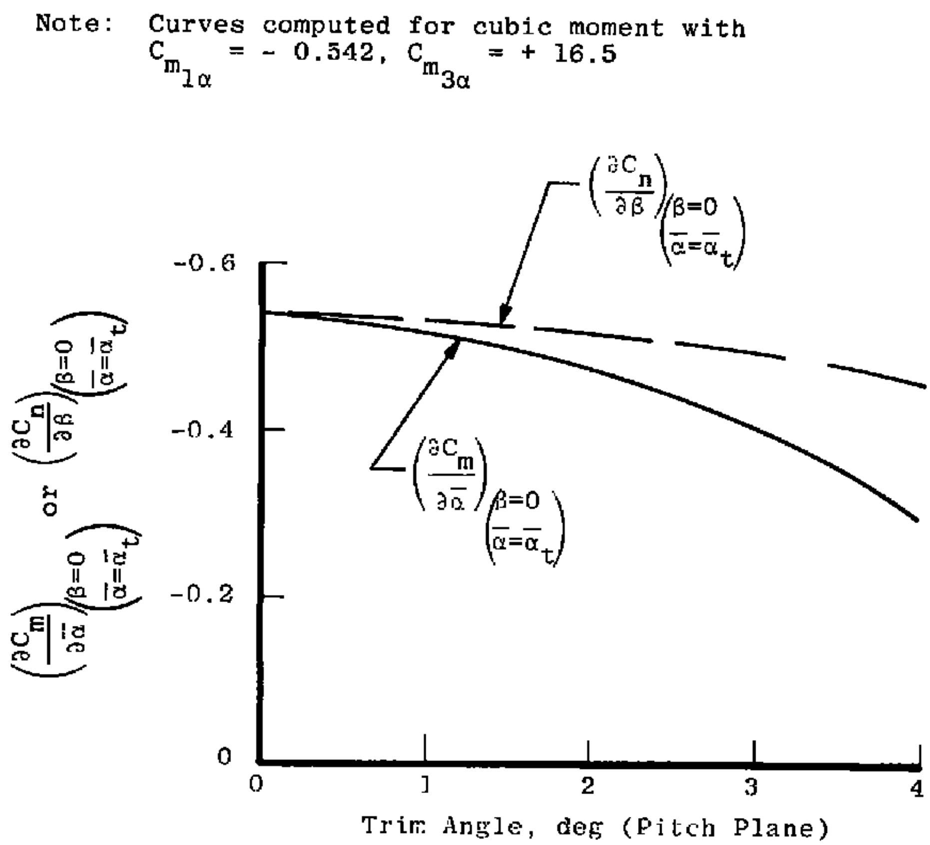


Figure 4. In-plane and out-of-plane moment data.

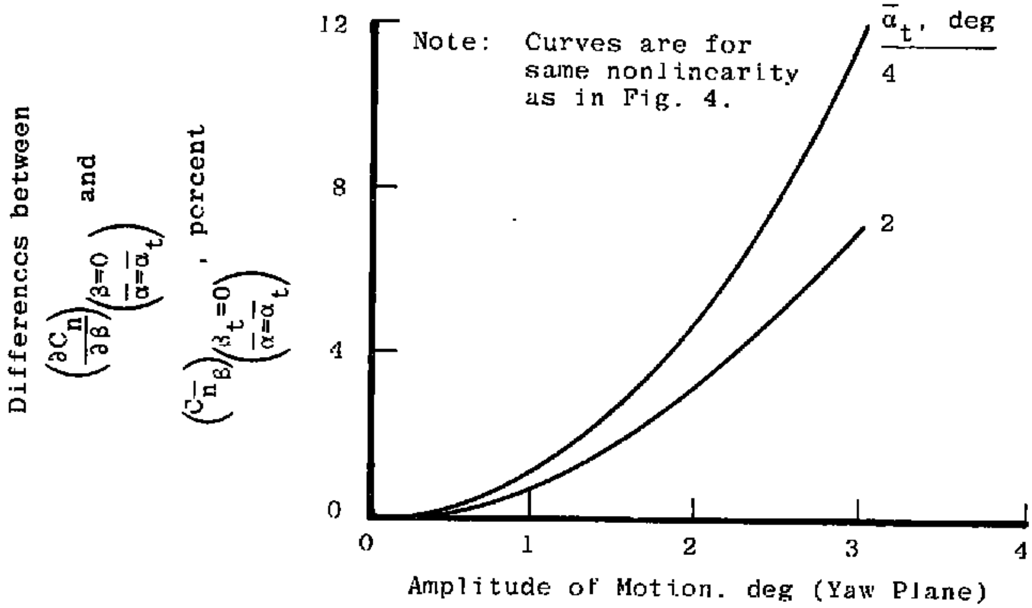


Figure 5. Comparisons of $C_{\bar{n}_\beta}$ and $\partial C_n/\partial \beta$ values.

CONCLUDING REMARKS

A study was made of the utilization of measurements of the effective moment derivative as obtained in small amplitude, large amplitude, and out-of-plane modes of testing. This study made use of the approximate relationship between a $C_{\bar{m}_\alpha}$ measurement and its parent nonlinear moment coefficient, C_{m_t}, as defined in Ref. 1. Results of the study indicate the following:

1. A nonlinear moment curve can be defined very well with $C_{\bar{m}_\alpha}$ measurements and corresponding effective angles of attack obtained in large amplitude tests.

2. In small amplitude testing, it is important always to minimize the motion amplitude if the measured effective parameter, $C_{\bar{m}_\alpha}$, is to adequately approximate the desired corresponding $\partial C_{m_t}/\partial \alpha$ parameter. This is true whether the small amplitude tests are for a zero or non-zero trim angle of attack. A nontrivial measuring error associated with the particular test equipment used could be an additional factor in selecting the optimum minimum test amplitude.

3. One must exercise care in assessing the usefulness of out-of-plane $C_{\bar{n}_\beta}$ measurements in the general case of an asymmetric body.

REFERENCES

1. Welsh, C. J., Lawrence, W. R., and Watt, R. M. "Real Gas Effects on the Aerodynamics of Blunt Cones as Measured in a Hypervelocity Range." AEDC-TR-79-33 (AD-A075526), October 1979.

2. Ward, L. K. and Uselton, B. L. "Dynamic Stability Results for Sharp and Blunted 10-Deg Cones at Hypersonic Speeds." AEDC-TR-68-277 (AD-845053), December 1968.

3. Clay, James T. "Nose Bluntness, Cone Angle, and Mach Number Effects on the Stability Derivatives of Slender Cones." Aerospace Research Laboratories ARL-67-0185 (AD661549), September 1967.

4. Uselton, James C., and Uselton, B. L. "A Look at the Validity of the Small Amplitude Oscillation Dynamic-Stability Measurement Technique." AIAA Paper 75-211, January 1975.

5. Uselton, Bob L. and Uselton, James C. "Test Mechanism for Measuring Pitch-Damping Derivatives of Missile Configurations at High Angles of Attack." AEDC-TR-75-43 (AD-A009865), May 1975.

6. Walchner, O. and Sawyer, F. M. "In-Plane and Out-of-Plane Stability Derivatives of Slender Cones at Mach 14." Aerospace Research Laboratories ARL 73-0090 (AD765164), July 1973.

7. Sawyer, Frank M. and Ingram, Charles W. "In-Plane and Out-of-Plane Stability Coefficients of a Slender Biconic Reentry Vehicle at Mach 14." Aerospace Research Laboratories ARL TR 74-0135, September 1974.

8. Uselton, Bob L., and Jenke, Leroy M. "Pitch- Yaw-, and Roll-Damping Characteristics of a Shuttle Orbiter at $M_\infty = 8$." AEDC TR-74-129 (AD-A011648), May 1975.

9. Murphy, C. H. "Free Flight Motion of Symmetric Missiles." BRL Report No. 1216, July 1963.

10. Rasmussen, Maurice L. and Kirk, Donn B. "On the Pitching and Yawing Motion of a Spinning Symmetric Missile Governed by an Arbitrary Nonlinear Restoring Moment." NASA TN D-2135, March 1964.

11. Morrison, A. M., Solomon, J. M., Ciment, M. and Ferguson, R. E. "Handbook of Inviscid Sphere-Cone Flow Fields and Pressure Distributions." Vol. II, NSWC/WOL/TR 75-45, December 1975.

NOMENCLATURE

C_{m_t}	Moment coefficient corresponding to the total angle-of-attack plane
C_m	Moment coefficient about the y-axis (see page 14)
C_n	Moment coefficient about the z-axis (see page 14)
C_{m_α}	Moment stability derivative
$C_{\bar{m}_\alpha}$	Effective moment parameter obtained in a linear fit of a nonlinear moment history
$C_{m_{1\alpha}}, C_{m_{3\alpha}}$	Coefficients for a cubic moment variation associated with the total angle-of-attack plane [see Eq. (2)]
$C_{m_{1\bar{\alpha}}}, C_{m_{3\bar{\alpha}}}$, $C_{m_{1\beta}}, C_{m_{3\beta}}$	Coefficients for a cubic moment variation — associated with y and z components of the angular motion [see Eqs. (7) and (8)]
α	Total angle-of-attack (small angles) or sine of the total angle of attack (large angles)
V	Total velocity vector
v	Velocity component along the y-axis
w	Velocity component along the z-axis
$\bar{\alpha}$, β	Components of α
α_e	Effective angle of attack, [see Eq. (3)]
α_t, $\bar{\alpha}_t$, β_t	Trim angles
θ_m	Average of (a) the midrange amplitude of the fitted interval of motion and (b) the mean of the initial and final motion amplitudes of the fitted interval

Note: All moment data are computed using the model diameter as the reference length and the model center of gravity as the moment reference.